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Longitudinal polarizability and the electronic topological transitions in a quasi-one-dimensional electron gas

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Abstract. The longitudinal polarization function of a quasi-one-dimensional electron gas (Q1DEG) confined in a semiconductor quantum well wire (QWW) is given in the random-phase approximation (RPA) for the cases where $T = 0$ K and $T \neq 0$ K, taking into account the lack of spatial invariance in the plane of the cross-section of the QWW. It is given as a sum of terms in the form of a spatial function, composed from the wavefunctions, and their complex conjugates, of two single-electron states and the polarizability for each allowed transition. The polarizability for a given transition is studied in terms of its frequency, momentum and temperature dependences, relating its behaviour with the electronic topological transition occurring in the Fermi domain. The results are analysed by representing the electron gas involved in the transition as composed from two quasi-particle subgases with different effective masses within the conduction band as an excitonic gas.

1. Introduction

The polarizability of the electron gas has been studied by numerous authors on account of its central importance in the study of many physical properties of the electron assembly. While exchange and correlation effects have been included for 3D systems in various approximations [1], the most common model used for 2D and, especially, 1D [2–13] is the RPA. The purpose of this paper is to stress some aspects of the 1D RPA polarizability not hitherto studied.

The analysis of the behaviour of the polarizability in low-dimensional electron gases is particularly important, because it allows us to study the appearance of charge-density waves (CDW) in systems like quasi-two-dimensional electron gases [14]. Some authors have suggested that the enhancement of the polarizability due to CDW instabilities is an attribute of the spatial geometry acquired by the Fermi domain in such Q2DEG. These instabilities provoke sharp peaks for values of the momentum transferred, q, equal to $2\kappa_F$, where κ_F is the 2D Fermi wavevector.

It was noted in a study of the specific heat of the Q1DEG in a QWW that this has a singularity of the λ-type [15]. In the present work this, as well as the behaviour of any other physical property depending on the polarizability, is traced back to the fact that the 1D case has a specific topological feature. This, which will be called a *one-dimensional*

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electronic topological transition (1DETT), has in turn direct implications for the physical properties depending on it.

The purpose of the present paper is to give an explicit formula for the longitudinal polarizability at $T = 0$ K which permits the analysis of the behaviour of the Q1DEG and its dependence on the 1DETT. Section 2 is devoted to the derivation of the longitudinal polarizability for the $T = 0$ K and $T \neq 0$ K cases. Section 3 includes an analysis of the dependence of the polarizability on the occurrence of a 1DETT, section 4 is devoted to the analysis of the numerical results and finally some conclusions are drawn.

2. Longitudinal polarizability in a Q1DEG

The function obtained through the calculation of the longitudinal dielectric response function—as was done in [16]—is understood as the longitudinal polarizability in a Q1DEG. However, we did our calculations using the inverse of the longitudinal dielectric response function in a Q1DEG.

The inverse dielectric function is the kernel resolvent of the Dyson equation for the screened electron–electron interaction energy taken in an approximation that is determined in the analysis. For the Q1DEG, for which there is only translational invariance in the z-direction, this Dyson equation can be written as

$$U_S(\omega, q; \boldsymbol{\rho}, \boldsymbol{\rho}') = U_0(q; \boldsymbol{\rho}, \boldsymbol{\rho}') + \int \mathrm{d}\boldsymbol{\rho}_1 \int \mathrm{d}\boldsymbol{\rho}_2 \; U_0(q; \boldsymbol{\rho}, \boldsymbol{\rho}_1)\Pi_0(\omega, q; \boldsymbol{\rho}_1, \boldsymbol{\rho}_2)U_S(\omega, q; \boldsymbol{\rho}_2, \boldsymbol{\rho}') \tag{1}$$

where $U_0(q; \boldsymbol{\rho}, \boldsymbol{\rho}')$ is the bare electron–electron interaction energy which can take into account the dielectric discontinuity in the system and $\Pi_0(\omega, q; \boldsymbol{\rho}_1, \boldsymbol{\rho}_2)$ is the irreducible polarization factor described by the bubble diagram in the RPA [1], given by

$$\Pi_0(\mathrm{i}\omega_p, q; \boldsymbol{\rho}, \boldsymbol{\rho}') = -\frac{1}{\beta}\sum_{\mathrm{i}\omega_r} \mathcal{G}(\mathrm{i}\omega_r, q; \boldsymbol{\rho}, \boldsymbol{\rho}')\mathcal{G}(\mathrm{i}\omega_r - \mathrm{i}\omega_p, q; \boldsymbol{\rho}, \boldsymbol{\rho}') \tag{2}$$

where $\boldsymbol{\rho} = (x, y)$ is the position vector in the cross-section plane of the QWW. Here the Matsubara Green function (MGF) for the Q1DEG is given by

$$\mathcal{G}(\mathrm{i}\omega_r, q; \boldsymbol{\rho}, \boldsymbol{\rho}') = \sum_{m,n} \frac{\phi_{m,n}(\boldsymbol{\rho})\phi_{m,n}(\boldsymbol{\rho}')}{\mathrm{i}\hbar\omega_r - E_{m,n}(q) + E_F} \tag{3}$$

in which the single-electron states are characterized by the dispersion relation $E_{m,n}(q)$ and by the wavefunction $\phi_{m,n}(\boldsymbol{\rho})$ which are the envelope functions of the conduction band taken, for instance, in the Hartree approximation or with exchange and correlation effects calculated self-consistently in the usual way, which uses the characteristics of the structure given by the matching conditions for the interfaces of the system. q represents the one-dimensional momentum transfer $k' - k$; E_F gives the Fermi level (chemical potential) measured from the bottom of the one-dimensional conduction subband for the electron gas.

Using (3) and (2) and performing the frequency summation as in [1] (for the case where $T = 0$ K, the frequency summation becomes an integral), we obtain, after analytical continuation in frequency has been carried out in the usual way [1],

$$\Pi_0(\omega, q, T; \boldsymbol{\rho}_1, \boldsymbol{\rho}_2) = \sum_{m,n;m',n'} \Pi_0(\omega, q, T; m, n; m', n')\phi^*_{m,n}(\boldsymbol{\rho})\phi_{m,n}(\boldsymbol{\rho}')\phi_{m',n'}(\boldsymbol{\rho})\phi^*_{m',n'}(\boldsymbol{\rho}') \tag{4}$$

which is the sum over all possible transitions in the system of terms in the form of the polarizability factor and a spatial function depending on four monoelectronic wavefunctions of the two states involved in a particular transition. The polarizability factor is given by

$$\Pi_0(\omega, q, T; m, n; m', n') = \frac{2}{L}\sum_k \frac{n_F(E_{m,n}(k)) - n_F(E_{m',n'}(k+q))}{\overline{h}\omega + \mathrm{i}\overline{h}\eta + E_{m,n}(k) - E_{m',n'}(k+q)}. \tag{5}$$

Here the factor 2 comes from the two possible projections of the spin; $n_F(E_{m,n}(k))$ is the Fermi–Dirac distribution function and L is a normalization length of the QWW in the z-direction.

The sum in (4) can be reorganized in the following way: consider the two terms involving the same states m, n and m', n'. Their spatial parts are the same and (4) can be rewritten as

$$\Pi_0(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) = \sum_{m,n,m',n'} R(n, m; n'm')\phi^*_{m,n}(\boldsymbol{\rho})\phi_{m,n}(\boldsymbol{\rho}')\phi_{m',n'}(\boldsymbol{\rho})\phi^*_{m',n'}(\boldsymbol{\rho}') \tag{6}$$

with

$$R(n, m; n', m') = \Pi_0(n, m; n', m') + \Pi_0(n', m'; n, m). \tag{7}$$

In equations (6) and (7) and from now on, the dependences on ω, q and T are everywhere understood.

Let us concentrate our attention on equation (7), i.e., the polarizability factor of one individual transition. As for a transition to take place one state must be empty and the other occupied, using (5) it is not difficult to see that at $T = 0$ K only one of the n_F-functions in the numerator will be zero. Reorganizing this sum, it can be written as

$$R(n, m; n'm') = P_0(m, n; m', n') + P_0(m', n'; m, n) \tag{8}$$

where

$$P_0(m, n; m', n') = \frac{2}{L}\sum_k \{H_1 - H_2\} \tag{9}$$

$$H_1 = \frac{n_F(E_{m,n}(k))}{\overline{h}\omega + \mathrm{i}\overline{h}\eta + E_{m,n}(k) - E_{m',n'}(k+q)}$$

$$H_2 = \frac{n_F(E_{m,n}(k))}{\overline{h}\omega + \mathrm{i}\overline{h}\eta + E_{m',n'}(k) - E_{m,n}(k+q)}.$$

In the second term of (9), as k is a summation variable, it can be changed as $k' \to k + q$ and then

$$P_0(m, n; m', n') = \frac{2}{L}\sum_k n_F(E_{m,n}(k))\{G_1 - G_2\} \tag{10}$$

$$G_1 = \frac{1}{\overline{h}\omega + \mathrm{i}\overline{h}\eta + E_{m,n}(k) - E_{m',n'}(k+q)}$$

$$G_2 = \frac{1}{\overline{h}\omega + \mathrm{i}\overline{h}\eta + E_{m',n'}(k-q) - E_{m,n}(k)}.$$

$P_0(n, m; n', m')$ is the longitudinal polarizability function which we aimed to derive in our calculations.

The polarization function in then given by (6) with each term in the sum formed by the polarizability given in (10) and the spatial function just described above.

3. The polarizability function in the case of electronic topological transition

Let us concentrate our attention on the polarizability given in (10) for a particular transition between two given monoelectronic states and let us describe its behaviour considering the possible 1DETT that may occur in the system.

We write the energies of the subbands (m, n) of a QWW as

$$E_{m,n}(k) = \frac{\overline{h}^2 k^2}{2m^*} + E_{m,n} \tag{11}$$

where k is the 1D wavevector and m, n label the quantized levels in 2D. The Fermi domain is then 1D and consists of segments in k-space. The number of such segments—often just one in practice—is the number of populated subbands.

We define the parameter $\Delta(m, n)$ as

$$\Delta(m, n) = E_F - E_{m,n}. \tag{12}$$

As shown elsewhere [15], the 1DETT takes place for $\Delta(m, n) = 0$, when E_F is at the critical level $E_{m,n}$. The specific heat, for instance, has a singularity of the λ-type in the vicinity of $\Delta(m, n) = 0$ [15].

Then, from (12) and (11), the polarizability (10) can be written as

$$P_0(n, m; n', m') = \frac{2}{L} \sum_k \left\{ \frac{n_F(E_{m,n}(k))}{A_1} - \frac{n_F(E_{m,n}(k))}{A_2} \right\} \tag{13}$$

$$A_1 = \overline{h}\omega + \mathrm{i}\overline{h}\eta + \left[\Delta(m, n) - \Delta(m', n')\right] - \frac{\overline{h}^2}{2m^*}\left[(k+q)^2 - k^2\right]$$

$$A_2 = \overline{h}\omega + \mathrm{i}\overline{h}\eta + \left[\Delta(m', n') - \Delta(m, n)\right] - \frac{\overline{h}^2}{2m^*}\left[(k-q)^2 - k^2\right].$$

Now, the electron–hole pair excitations have in the 1D case a specific topological feature in that every excitation, by producing a hole in the Fermi domain, changes its connectivity, as it disconnects two fragments of a segment in k-space. This has physical implications, as will be seen presently.

For the evaluation of (13) we transform the sum into an integral in the usual way. For $T = 0$ K the integration can be carried out analytically; one thus obtains for the real part of the polarizability

$$\Re\{P_0(\omega, q, 0; \mu)\} = \frac{m^*}{\pi\overline{h}^2 q} \log\left|\frac{(\overline{h}\omega)^2 - (E_\mu^{(-)}(q))^2}{(\overline{h}\omega)^2 - (E_\mu^{(+)}(q))^2}\right| \tag{14}$$

and for the imaginary part

$$\Im\{P_0(\omega, q, 0; \mu)\} = \frac{m^*}{\pi\overline{h}^2 q}\left\{ \arctan\left[\frac{2\overline{h}\eta\,\overline{h}\omega}{(\overline{h}\omega)^2 - (E_\mu^{(-)}(q))^2}\right] - \arctan\left[\frac{2\overline{h}\eta\,\overline{h}\omega}{(\overline{h}\omega)^2 - (E_\mu^{(+)}(q))^2}\right]\right\} \tag{15}$$

where μ represents the combination of indices $(n, m; n', m')$. To obtain (14) and (15), we considered only linear terms in $\overline{h}\eta$, η being an infinitesimal frequency. Here we have put

$$E_\mu^{(\pm)}(q) = \Delta(m', n') - \Delta(m, n) + \frac{\overline{h}^2 q^2}{2m^*}\left[1 \pm \frac{2k_{E_F}}{q}\right] \tag{16}$$

where k_{E_F} is the one-dimensional Fermi wavevector; q could reach values in the range $-2k_F \leqslant q \leqslant 2k_F$. Equation (14) is the same as that obtained in [10] for $\overline{h}\omega = 0$.

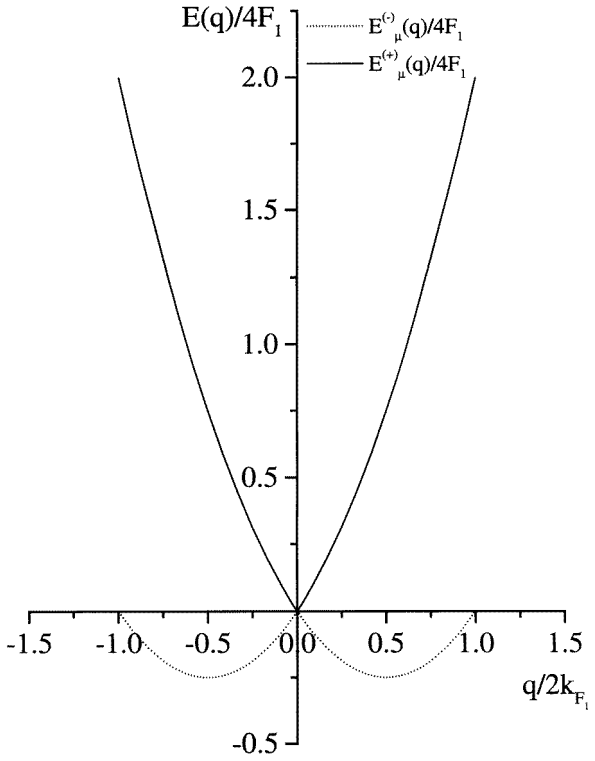

Figure 1. A graphical representation of the dispersion equations for 'light particles' (E) (continuous line) and 'heavy particles' (H) (dashed line); $\Delta(m', n') - \Delta(m, n) = 0$ is assumed in equation (16). $F_1 \equiv E_F$ and $k_{F_1} \equiv k_F$ are valid for all of the figures.

Here (16) can be interpreted as follows: the electron gas involved in the transition from subband m, n to subband m', n' behaves as if composed of two gases of quasi-particles, one of which we call 'light' (corresponding to the $+$ sign) and will also call a quasi-electron and represent by the letter E, and the other 'heavy' (corresponding to the $-$ sign) which we will also call a quasi-hole and represent by the letter H. Then (16) are the dispersion relations for the quasi-particles E and H corresponding to the gas involved in the transition $m, n \rightarrow m', n'$.

For $T \neq 0$ K an analytical expression for P_0 cannot be obtained; however, expressions (16) still appear in the longitudinal polarizability function, with the same interpretation.

Then, like all of the collective properties of the system, the polarizability is dominated by the geometry of the Fermi domain and the changes in its topology determine the behaviour of the property.

4. Results and discussion

We shall assume that from the outset there is a void in the Fermi segment due to a Fermi energy above, at least, its first critical value E_{11}; this is represented by E_0 in all of the following figures. We normalize all of the energy functions with respect to E_0, so the following results are independent of the cross-section geometry and the kind of material of

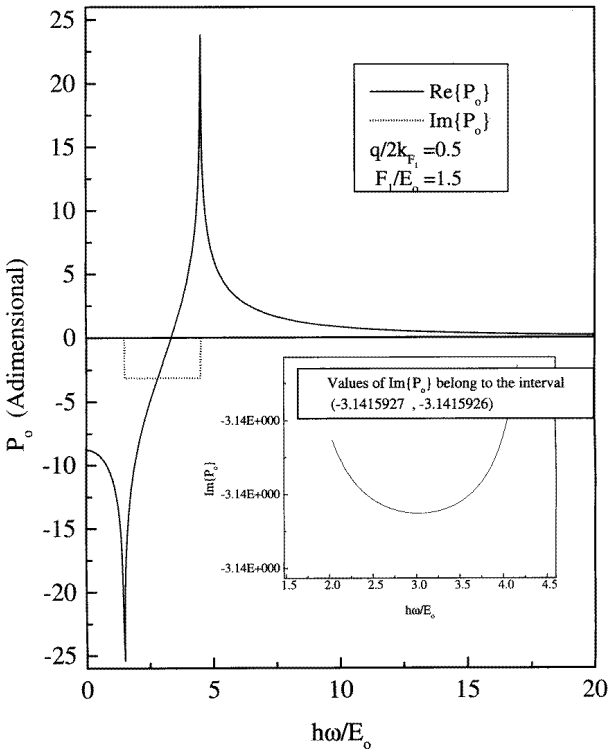

Figure 2. Adimensional real and imaginary parts of the longitudinal polarizability function $P_0(\omega, q, T; m, n; m', n')$ versus $\overline{h}\omega/E_0$; $T = 0$ K is assumed in the calculations. One can see in the inset the existence of a minimum for $\Im\{P_0\}$ inside a very small interval of $\overline{h}\omega/E_0$.

which the semiconductor QWW is composed if just intrasubband electronic transitions are considered. For intersubband ones, it is necessary to take account of the geometry of the cross-section of the wire. Our analysis starts with the $T = 0$ K case.

According to the definition of q, equation (16) represents the dispersion equations for particles E and H which can be an electron (E) and a hole (H) at the conduction band (see figure 1). This result was obtained under the assumption that $\Delta(m', n') - \Delta(m, n) = 0$, i.e. only intrasubband transitions are considered due to the electron–electron interaction. As a result, we observe a Coulombic gap at the conduction band due to the electron–electron interaction in the QWW. The energy levels corresponding to a transition appearing in (10) belong to coupled states of E and H in the same way as for the exciton formation from the conduction band and the valence band in a bulk semiconductor. An external action of energy $\overline{h}\omega$, for a fixed value of q, provokes the excitation of one or the other kind of quasi-particle, changing the behaviour of the polarizability of this particular gas (see figure 2). Here it is also assumed that only intrasubband electronic transitions occur.

As $\overline{h}\omega$ increases from zero to $E_\mu^{(-)}(q)$, mainly the H-particles provide a screening from the external action, i.e., provide a polarizability in opposition to the external action. This behaviour is inferred from the sign of (14) in this range. At the same time, a small negative imaginary part of the polarization occurs, as seen from (15). For $\overline{h}\omega = E_\mu^{(-)}(q)$, a resonance occurs in transitions corresponding to the H-particles and the polarizability has a negative maximum value. For $E_\mu^{(-)}(q) < \overline{h}\omega < E_\mu^{(+)}(q)$, the real part of the polarizability changes

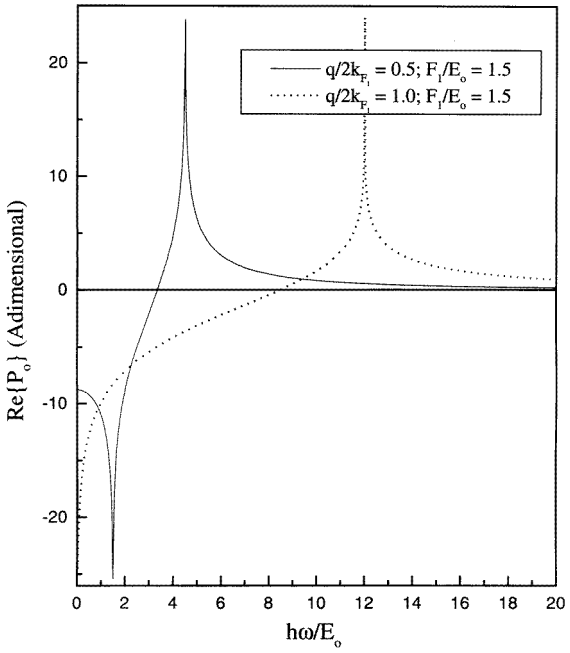

Figure 3. The adimensional real part of P_0 versus $\hbar\omega/E_0$ for different values of $q/2k_F$.

its sign at the value

$$\hbar\omega_b = \sqrt{(E_\mu^{(+)}(q))^2 + (E_\mu^{(-)}(q))^2}. \tag{17}$$

For $\hbar\omega_b < \hbar\omega < E_\mu^{(+)}(q)$, the excitation of this electronic gas is dominated by the E-particles which provide a positive real part, i.e. a reinforcement of the external action, with an imaginary part that is always negative.

For $\hbar\omega = E_\mu^{(+)}(q)$, the E-particles give the maximum reinforcement to the external action. For $\hbar\omega > E_\mu^{(+)}(q)$, both the real and the imaginary parts of the polarizability tend to zero in accordance with the fact that at this frequency the electronic gas cannot follow the external action.

From equation (9) it can be seen that the imaginary part of the polarizability does not change sign throughout the energy interval. Physically, this can be explained as a consequence of the fact that only the electron gas can receive energy from the external agent and energy cannot be produced due to its action. It has a negative sign and it can be shown that it has a minimum at

$$\hbar\omega_a = \frac{1}{\sqrt{6}}\sqrt{E_T + \sqrt{E_T^2 + 12(E_\mu^{(+)}(q)E_\mu^{(-)}(q))^2}} \tag{18}$$
$$E_T = (E_\mu^{(+)}(q) + E_\mu^{(-)}(q))^2.$$

This behaviour of $P_0(\omega, q, 0; \mu)$, although easy to obtain and clear to see, does not correspond to the macroscopic behaviour of the whole electronic gas because one has to sum the contributions from all possible transitions allowed in the system to get the total response; then when one particular gas (corresponding to the transition $\mu_1 \Rightarrow m_1, n_1, m_1', n_1'$) has a screening behaviour (i.e. opposite to the external action), some other transitions will

contribute in the same way but others could exhibit reinforcement behaviour. Then, the whole Q1DEG could have any behaviour macroscopically and the behaviour cannot be predicted before the numerical calculation is done.

In figure 3 we can observe the behaviour of the polarizability versus $\hbar\omega$ for two different values of q. Changes in q cause a change in the values of $\hbar\omega$ for which resonance occurs. For $0 < q < k_F$, the resonance due to H-particles occurs for values of the energy less than E_F. At $q = k_F$, the resonance of heavy particles occurs exactly at E_F. For $k_F < q < 2k_F$, this resonance is reduced again to energies less than E_F and will occur at $\hbar\omega = 0$ when $q = 2k_F$, and starts to move to higher energies for $q > 2k_F$. At the same time, the resonance of E-particles always moves to higher energies as q increases. Then, for small values of q the 'boiling' of the electronic gas is dominated by one half of the Fermi segment (the Fermi void) for small energies and for big values of q it is dominated by the other half of the Fermi segment. This behaviour can be seen as follows: the holes H are the contribution of electrons that belong to states closer at the edge of the Fermi segment. As q increases, electrons of states deeper in the Fermi segment could participate in the transition, i.e., the states approaching the centre of the segment. At $q = k_F$, the electron at the centre of the segment can participate in the transition. For $k_F < q < 2k_F$, the electrons of the other half of the Fermi segment could participate in the transition, receiving a great momentum in the opposite direction to the one they are travelling in due to the external action, and they dominate the response. The screening is in the opposite direction from the direction of travel of the electrons belonging to the other half of the Fermi segment at high energies. Here, as in figure 1 and figure 2, only intrasubband electronic transitions are considered.

In figure 4 we show the temperature dependence of the polarizability function. Here it is plotted as the real part of the polarizability ($\Re\{P_0\}$) versus $\hbar\omega$ for different values of

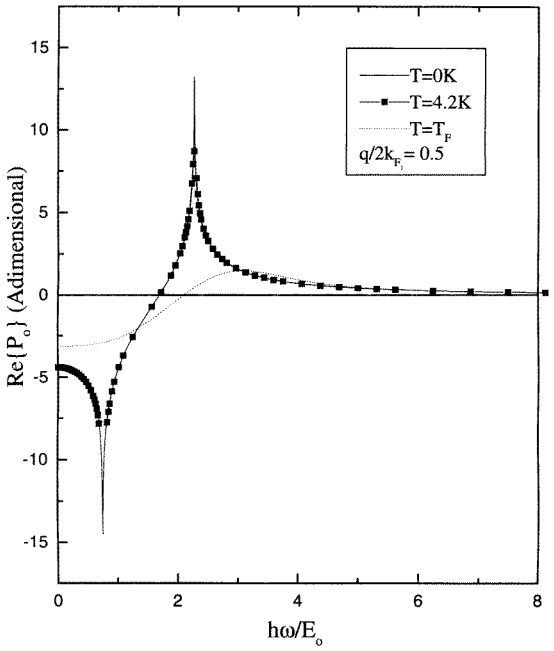

Figure 4. The temperature dependence of the adimensional real part of P_0; the temperatures selected were 0 K, 4.2 K and the Fermi temperature of the Q1DEG. E_F/E_0 was taken as 1.76.

T. As can be observed, the increments of the temperature provoke a smoothness of the logarithmic resonant effects in the polarizability function without causing any qualitative change in its behaviour with respect to $\overline{h}\omega$ for small enough temperatures. For values of the temperature greater than 10 K, a displacement of $\overline{h}\omega_b$ occurs (to higher values), whereupon the real part of the polarizability changes its sign. Nevertheless, the coexistence of the two kinds of quasi-particle is still occurring. This behaviour is in perfect agreement with the temperature characteristic of the electron distribution function. This result is independent of the value of $q/2k_F$, i.e. independent of the energy position of the electron inside the Fermi segment. In figure 4, $q/2k_F = 0.5$ and $E_F/E_0 = 1.76$ were taken on the assumption of an intrasubband electron–electron scattering mechanism.

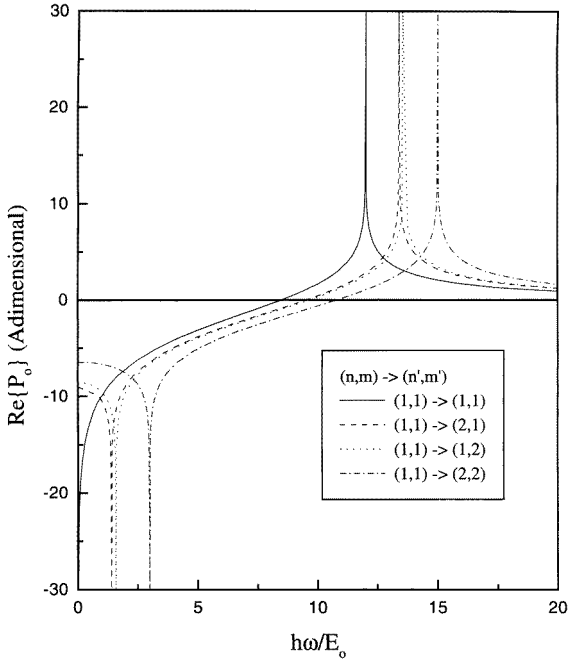

Figure 5. The electronic transition dependence of the adimensional real part of P_0 versus $\overline{h}\omega/E_0$. $T = 0$ K; $q/2k_F = 1.0$ and $E_F/E_0 = 1.5$. The shifts of the curves represent the kind of transition that has been realized.

In figure 5 we show the adimensional real part of P_0 versus $\overline{h}\omega/E_0$ in the cases of intra- and intersubband electronic transitions due to electron–electron interaction. We present the results for $T = 0$ K; however, for $T > 0$ K the situation is similar. As was remarked at the beginning of this section, for the case of intersubband transitions it is necessary to take account of the cross-section geometry of the QWW. For practical purposes, we assume a rectangular cross-section with an infinite potential energy barrier at the frontiers of the QWW which simplify the expressions in the calculation. The intra- and intersubband electronic transitions were considered as coming from the basic energy level $E_{11} = E_0$ up to higher levels, as $E_F/E_0 = 1.5$, which means that only the basic level is occupied. Using the curve for the transition $(1, 1) \rightarrow (1, 1)$ as a reference, the other curves for different transitions shift to higher energies. This means that the external action must supply more energy, for the same value of q, for the same polarizability effect to be achieved for transitions to higher levels.

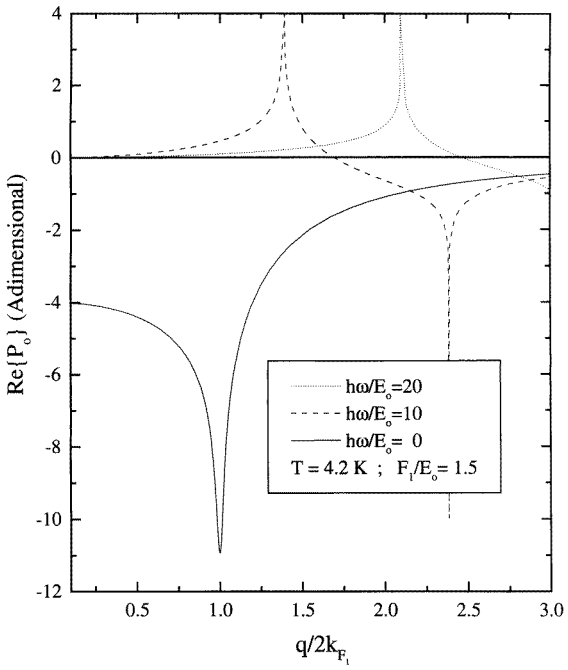

Figure 6. The real part of P_0 versus $q/2k_F$. The temperature was taken as 4.2 K, but for $T = 0$ K the result is similar.

Figure 6 shows the real part of P_0 versus q for fixed values of the external energy $\overline{h}\omega$ given to the Q1DEG. The figure is obtained for $T = 4.2$ K, but the behaviour is practically the same as that for the $T = 0$ K case except for the sharpness of the singularities. The most significant fact is the result for $\overline{h}\omega = 0$, i.e., the static case. In this case there is only evidence of existence of H-particles with negative $\Re\{P_0\}$; this shows that in the absence of an external action only the H-particles give a screening in the Q1DEG. For $q = 2k_F$, a logarithmic singularity is produced which is the well known Fröhlich–Peierls transition that is infinite for $T = 0$ K and smoothed for $T > 0$ K. If we follow L J Sham [18], the Peierls transition is an electron–phonon interaction effect in Q1DEG due to the aperture of the Coulombic gap in the conduction band [19].

In [10] it was shown that this singularity is also obtained taking into account only the electron–ionized impurity interaction for a Q1DEG. Now, in this paper, taking only the electron–electron interaction into account, we reproduce the Fröhlich–Peierls singularity. From all of this, we can conclude that this effect is due just to the dimensionality condition of the system, which provokes the appearance of a 'gap' in the conduction band, and not by any other particular interaction considered. Again, here we considered only intrasubband electronic transitions for the electron–electron scattering mechanism. However, for intersubband transitions the results are the same.

The one-dimensional electronic topological transitions (1DETT) have been presented as lying behind all of the results mentioned before. For intra- or intersubband electronic transitions, P_0 depends on the Fermi level E_F. This fact could give us the relation between the Fermi void and any value of critical energy below it. Let us select E_0 for the sake of comparison. In figure 7 we show the adimensional real part of P_0 versus E_F/E_0 for different values of $q/2k_F$. We took the $T = 0$ K case for the calculations, but for the

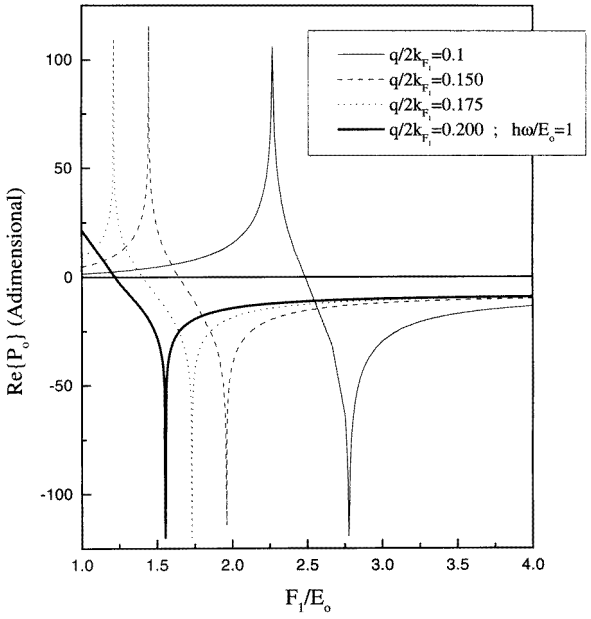

Figure 7. The real part of P_0 versus E_F/E_0 for different values of $q/2k_F$. The external action was considered energetically as $\overline{h}\omega/E_0 = 1$ and the temperature was taken as $T = 0$ K.

$T > 0$ K case the results are quite similar. As a relevant fact, we might mention that for a fixed $q/2k_F$ (even as k_F varies when E_F varies), this represents electrons which occupy certain positions inside the Fermi segment. Hence, when E_F increases, the electrons E and the holes H become excited only in a particular Fermi segment. For very deep electrons, when E_F increases the effect of polarizability is balanced by both subgases and only for particular values of E_F will one of them dominate over the other. Beside this, the behaviour of $\Re\{P_0\}$ shown in figure 7 is similar to that mentioned in the former analysis; it is due, in this case, to the enhancement of the Fermi void. The electronic resonance effect, even in the intrasubband case, will depend on the electronic topological transitions which have occurred.

5. Conclusions

The behaviour of the polarizability factor in a Q1DEG was studied and related to the occurrence of a 1DETT. The simulation of the electronic gas involved in the transition, studied as being composed of two quasi-particle subgases, was addressed; this explains qualitatively the behaviour of the polarizability when one changes the parameters of the system.

We emphasized that the polarizability behaviour is only indirectly tested in all of the measurements, due to the existence of several subbands which participate in the 'boiling' of the electronic gas when an external action is present. It is necessary to take into account the whole energy spectrum of single-electron states when considering the macroscopic effects of the external action on the gas, and the polarizabilities of different transitions will contribute, each in its particular way, to the screening of the external action.

There is in progress work studying the inverse dielectric function of a Q1DEG in

a semiconductor QWW, considering this behaviour of the polarizability for the allowed transitions.

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